Welfare Cost of Risk

(Boadway and Bruce, page 229)

Start with the expected utility function: $E(u) = \sum_s \pi_s \, v(p^s, m^s)$

where $\sum_s \pi_s = 1$ and v (   ) is the indirect utility function.

Assume that V (p, m) = f [e (p, u)]. This assumption provides a way to translate units of income into units of utility. This assumption was not needed in the measurement of welfare change under certainty.

Now the expected utility function can be written in terms of the expenditure function:

$$E(u) = \sum_s \pi_s \, f(e[p, u])$$

To measure the welfare cost of <u>income variability</u> assume the consumer is faced with a certain income of e^2, or an uncertain income. The probability of income e^1 ($< e^2$) is π_1; the probability of income e^3 ($> e^2$) is π_3. Finally, the expected value of the uncertain income stream is the same as the certain income:

$$\pi_1 \, e^1 + \pi_3 \, e^3 = e^2$$

The difference in the utility value of the certain and uncertain incomes is measured as follows:

$$\Delta E(u) = \left[\pi_1 \, f(e^1) + \pi_3 \, f(e^3)\right] - f(e^2)$$

Express f (e^1) and f (e^2) as Taylor's series' expansions around e^2:

$$\begin{aligned}
\Delta E(u) = {} & \pi_1\left[f(e^2) + f'(e^2)(e^1 - e^2) + (\tfrac{1}{2}) f''(e^2)(e^1 - e^2)^2 + R_1\right] \\
& + \pi_3\left[f(e^2) + f'(e^2)(e^3 - e^2) + \tfrac{1}{2} f''(e^2)(e^3 - e^2)^2 + R_3\right] - f(e^2) \\
= {} & \pi_1 \, f(e^2) + \pi_3 \, f(e^2) - f(e^2) \\
& + f'(e^2)\left[\pi_1 (e^1 - e^2) + \pi_3 (e^3 - e^2)\right] \\
& + (\tfrac{1}{2}) f''(e^2)\left[\pi_1 (e^1 - e^2)^2 + \pi_3 (e^3 - e^2)^2\right] + R_1 + R_3
\end{aligned}$$

Ignoring the residual terms, R_1 and R_3

$$\Delta\ E(u) \approx f'\left(e^2\right)\left[\pi_1\ e^1 + \pi_3\ e^3 - (\pi_1 + \pi_3)\ e^2\right]$$
$$+ \left(\tfrac{1}{2}\right) f''\left(e^2\right)\left(\text{var } e^2\right)$$
$$= f'\left(e^2\right)\left(e^2 - e^2\right) + \left(\tfrac{1}{2}\right) f''\left(e^2\right)\left(\text{var } e^2\right)$$
$$= \left(\tfrac{1}{2}\right) f''\left(e^2\right)\left(\text{var } e^2\right)$$

If $f'' < 0$ (risk aversion), the value of $\Delta\ E(U)$ will be negative. The consumer is worse off with variable income than with a certain income of the same expected value.

This measure is still in units of utility. To convert to dollars, we need to divide by the marginal utility of income (df/de). The welfare estimate becomes

$$\Delta W = \frac{\Delta E(u)}{f'\left(e^2\right)} \approx \left(\tfrac{1}{2}\right)\frac{f''\left(e^2\right)}{f'\left(e^2\right)}\left(\text{var } e^2\right).$$

The welfare loss is thus determined by half the absolute risk aversion coefficient times the variance of income.