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AREC 514

Theory of the Second Best

Assume a social welfare function U $(x^1, x^2 .... x^n)$ and a production function G $(x^1, x^2, ..., x^n) = 0$. In this formulation, the x^i represent both inputs (as negative numbers) and outputs (as positive numbers). A normal production function, $Q = F(Z^i)$, can be written as $Q - F(Z^i) = 0$, for example. In perfect competition with no divergences, society sets out to

$$\max \quad U(x^i)$$
$$\text{s.t.} \quad G(x^i) = 0$$

Form the Lagrangean, $L = U(x^i) - \lambda G(x^i)$. Maximize. First-order conditions are

$$\frac{\partial L}{\partial x^i} = 0 = \frac{\partial U}{\partial x^i} + \lambda \frac{\partial G}{\partial x^i}. \qquad \text{Let} \quad U_i \equiv \frac{\partial U}{\partial x^i}.$$

Then the first-order condition for x^i is $-U_i = \lambda G_i$. Also, $-U_n = \lambda G_n$. Therefore, we can describe the optimum conditions as

$$\frac{U_i}{U_n} = \frac{G_i}{G_n}.$$

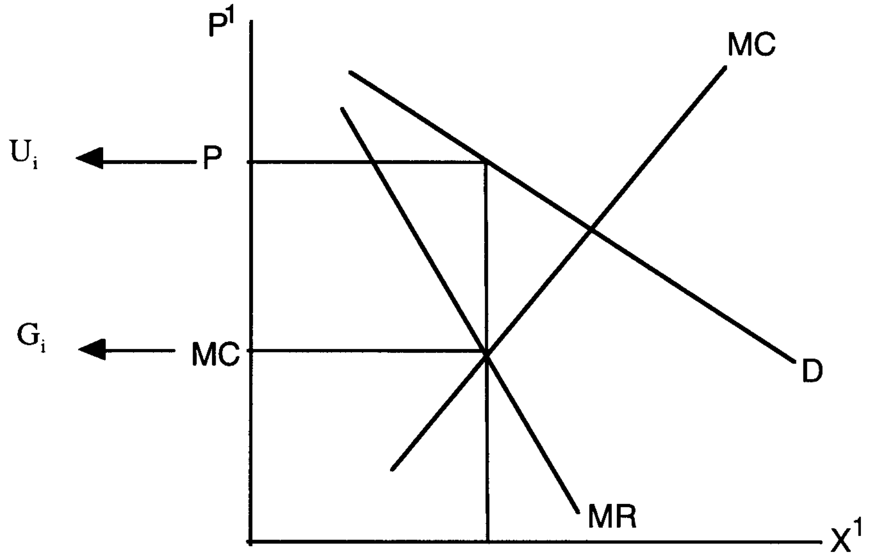
These ratios represent the <u>social</u> marginal rate of substitution and the marginal rate of transformation between x^i and x^n. Then we have

$$\frac{\partial U_i}{\partial U_n} = \frac{P^i}{P^n}, \quad \text{and} \quad \frac{G^i}{G^n} = \frac{MC^i}{MC^n} \quad \text{or} \quad \frac{P^i}{MC^i} = \frac{P^n}{MC^n} \; (= 1).$$

Now add a second constraint to the problem, of the form

$$\frac{U_1}{U_n} = k\frac{G_1}{G_n}, \; k \neq 1.$$

Such as constraint could arise with monopolistic production of good 1, for example.



Because of monopoly, marginal social utility from consumption exceeds marginal cost of production. Put another way $U_1 = k\,G_1$, $k>1$.

The welfare maximization problem becomes

$$\max \quad U(x^i)$$

$$\text{s.t.} \quad G(x^i) = 0$$

$$\underline{\text{and}} \quad \frac{U_1}{U_n} = \frac{k\,G_1}{G_n}.$$

The Lagrangean becomes

$$L = U(x^i) + \lambda\, G^i + \gamma\ (U_1/U_n - k\, G_1/G_n).$$

Maximize, determining the first-order conditions

$$\frac{\partial L}{\partial x^i} = 0 = U_i - \lambda G_i - \gamma \left(\frac{U_n U_{1i} - U_1 U_{ni}}{(U_n)^2} - k\,\frac{G_n G_{1i} - G_1 G_{ni}}{(G_n)^2} \right) \qquad (1)$$

Remember, by the rules of differentiation, that

$$\frac{\partial}{\partial x^i}\left[\frac{\partial U/\partial x^1}{\partial U/\partial x^n}\right] = \frac{\frac{\partial U}{\partial x^n}\frac{\partial^2 U}{\partial x^1\,\partial x^i} - \frac{\partial U}{\partial x^1}\frac{\partial^2 U}{\partial x^n\,\partial x^i}}{\left(\frac{\partial U}{\partial x^n}\right)^2} = \frac{U_n U_{1i} - U_1 U_{ni}}{U_n^2}$$

Thus, at the optimum,

$$-U_i = \lambda G_i + \gamma \qquad (\qquad\qquad\qquad)$$

$$or \quad -U_i = \lambda G_i\left(1+\frac{\gamma}{\lambda G_i}\qquad(\qquad\qquad\qquad)\right).$$

parentheses include the term in parentheses from equation 1.

Optimum conditions become

$$\frac{U_i}{U_n} = \frac{G_i}{G_n}\cdot\left[\frac{1+\dfrac{\gamma}{\lambda G_i}\left(\dfrac{U_n U_{1i} - U_1 U_{ni}}{U_n^2} - k\,\dfrac{G_n G_{1i} - G_1 G_{ni}}{G_n^2}\right)}{1+\dfrac{\gamma}{\lambda G_n}\left(\dfrac{U_n U_{1n} - U_1 U_{nn}}{U_n^2} - k\,\dfrac{G_n G_{1n} - G_1 G_{nn}}{G_n^2}\right)}\right].$$

If the term in brackets is different from 1, then the optimum second-best conditions will be different from the optimum first-best conditions, $U_i/U_n = G_i/G_n$. Under what circumstances will the bracketed term equal 1? If

$$\frac{\gamma}{\lambda G_i}\left(\frac{U_n U_{1i} - U_1 U_{ni}}{U_n^2} - k\,\frac{G_n G_{1i} - G_1 G_{ni}}{G_n^2}\right) = \frac{\gamma}{\lambda G_n}\left(\frac{U_n U_{1n} - U_1 U_{nn}}{U_n^2} - k\,\frac{G_n G_{1n} - G_1 G_{nn}}{G_n^2}\right)$$

$$\text{or} \quad \frac{1}{G_i}[\qquad\qquad\qquad\qquad] = \frac{1}{G_n}[\qquad\qquad\qquad\qquad].$$

In general, this will not occur for <u>any</u> pairwise comparisons. But for the optimum first-best conditions to hold, the above equality must prevail for <u>all</u> pairwise comparisons. The only condition that guarantees satisfaction of these circumstances is <u>separability.</u>

$$U = f^1(x_1) + f^2(x^2) + f^3(x^3) + \ldots + f^n(x^n)$$

and

$$G = h^1(x_1) + h_2(x^2) + h^3(x^3) + \ldots + h^n(x^n).$$

In this case, all second-derivatives (G_{1i}, U_{1i}, etc.) are <u>zero</u>. These conditions imply that commodity substitution in consumption is absent, and that input substitution in production is absent.

Otherwise, we are a second-best world. We cannot be sure removing divergences causes welfare increases.