

Risk in Public CBA

Two arguments have been raised that allow analysts to ignore risk in the valuation of public investment project values, and to calculate net benefits on the basis of expected values.

Risk Pooling

Assume there are n investment projects in the economy. For each project, the share of total investment cost is $\frac{1}{n}$, and the net benefit is Y_i .

The variance and covariance for the net benefits of project i are as follows:

$$\text{var}(Y_i) = E[Y_i - E(Y_i)]^2$$

$$\begin{aligned} \text{cov}(Y_i, Y_j) &= E[(Y_i - E(Y_i)) (Y_j - E(Y_j))] \\ &= 0 \end{aligned}$$

The variance of net benefits for all projects in the economy is as follows:

$$\begin{aligned} \text{var} \left(\sum_i Y_i \right) &= E \left[\sum_i \frac{1}{n} Y_i - \sum_i \frac{1}{n} E(Y_i) \right]^2 \\ &= \sum_i \frac{1}{n^2} [Y_i - E(Y_i)]^2 + 2 \sum_i \sum_j \frac{1}{n^2} \text{cov}(Y_i, Y_j) \\ &= \frac{1}{n^2} n (\text{var } Y_i) + (2) \left(\frac{1}{n^2} \right) (0) \\ &= \frac{1}{n} \text{var } Y_i \end{aligned}$$

$$\text{As } n \rightarrow \infty, \quad \text{var} \left(\sum_i Y_i \right) \rightarrow 0.$$

The law of large numbers that allows the direct cost of risk from a project to be ignored. Unexpectedly low net benefits in one project will be offset by unexpectedly high net benefits in another project.

Risk Spreading

In this case, the net benefits of project (Y_i) are assumed to be shared among m individuals.

Recall from an earlier handout that the welfare cost of risk (k) can be written as follows:

$$k = -\frac{1}{2} \frac{u''}{u'} \text{ var } \left(\frac{Y_i}{m} \right),$$

where $\frac{u''}{u'}$ = absolute risk aversion coefficient

and $\frac{Y_i}{m}$ = individual's share of project benefits.

The $\text{var} \left(\frac{Y_i}{m} \right)$ can be rewritten:

$$\text{var} \left(\frac{Y_i}{m} \right) = E \left(\frac{Y_i}{m} - \frac{E(Y_i)}{m} \right)^2 = \frac{\text{var } Y_i}{m^2}$$

Substituting this expression in the formula for the welfare cost of risk:

$$k = -\frac{1}{2} \frac{u''}{u'} \frac{\text{var } Y_i}{m^2}$$

As the population of the economy increases ($m \rightarrow \infty$), $k \rightarrow 0$. This is the Arrow-Lind Theorem. If risk is shared by a large number of persons, the risk of a project is negligible and can be ignored in the social perspective.