Risk Aversion Coefficients

Consider an individual with wealth Y. He is offered a bet that involves winning or losing an amount h. What are the probabilities p that can induce the individual to take the bet? We are looking for solution to the following equation:

$$U(Y) = p\left[U(Y+h)\right] + (1-p)\left[U(Y-h)\right] \qquad (1)$$

Expand U (Y + h) and U (Y - h) as Taylor's series expansions:

$$U(y+h) = U(Y) + h\,U'(Y) + {}^1/_2\, h^2\, U''(Y) + R_1 \qquad (2)$$

$$U(Y-h) = U(Y) - h\,U'(Y) + {}^1/_2\, h^2\, U''(Y) + R_2 \qquad (3)$$

Substitute (2) and (3) into (1), and ignore the remainders R_1 and R_2.

$$\begin{aligned} U(Y) &= p\,[U(Y) + h\,U'(Y) + {}^1/_2\, h^2 U''(Y)] \\ &\quad + (1-p)\,[U(Y) - h\,U'(Y) + {}^1/_2\, h^2 U''(Y)] \\ &= U(Y) + ph\,U'(Y) - (1-p)\,h\,U'(Y) \\ &\quad + p\,{}^1/_2\, h^2 (U''(Y)) + {}^1/_2 (1-p)(h^2 U''(Y)) \\ &= U(Y) + U'(Y)\,[2ph - h] + U''(Y)\left[\frac{ph^2}{2} + \frac{h^2}{2} - \frac{ph^2}{2}\right] \end{aligned}$$

$$U(Y) = U(Y) + U'(Y)\,[2ph - h] + U''(Y)\left[\frac{h^2}{2}\right]$$

$$O = U'(Y)\,[2p - 1]\,h + U''(Y)\left[\frac{h^2}{2}\right]$$

$$-U''(Y)\left[\frac{h^2}{2}\right] = U'(Y)\,[2p - 1]h$$

$$-\frac{U''(Y)}{U'(Y)}\,\frac{h}{2} = 2p - 1$$

$$\frac{-U''(Y)}{U'(Y)} \frac{h}{4} + \frac{1}{2} = p$$

$$p = \frac{1}{2} + \frac{h}{4} \frac{-U''(Y)}{U'(Y)} \tag{4}$$

So absolute risk aversion measures the insistence of an individual for more-than-fair odds (one-half plus the term in equation (4)). Notice that the acceptance probability is also related to the size of the bet, h. Smaller bets require smaller premium for the probability.

If we measure the bet in proportion to Y, and let $n = h/y$ then equation (4) becomes

$$p = \frac{1}{2} + \frac{n}{4} \frac{(-Y)(U''(Y))}{U'(Y)} \tag{5}$$

Now the probability premium is expressed in terms of the income share (n), and the relative risk aversion coefficient, $((-Y)(U''(Y)/U'(Y))$.