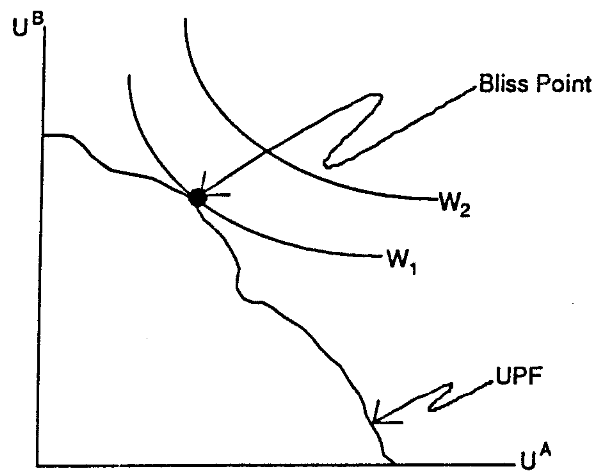
Optimum Conditions in the Bergson-Samuelson Social Welfare Function



The slope of the utility possibilities frontier (UPF) is dU_B/dU_A (<0). The slope of the indifference contour of the social welfare function ($W = F(U^A(x_1, \ldots x_n), U^B(x_1, \ldots x_n))$ is $(\partial W/\partial U^B)/(\partial W/\partial U^A)$. At maximum social welfare, or the bliss point, an indifference curve will be tangent to the UPF, so that

$$\frac{dU^B}{dU^A} = \frac{\partial W/\partial U^B}{\partial W/\partial U^A} \qquad (1)$$

What are the characteristics of this bliss point? To see these characteristics, begin by differentiating the individual's utility function:

$$dU = \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 \qquad (2)$$

Rearranging,

$$\frac{du}{\partial U/\partial x_2} = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} dx_1 + dx_2 \qquad (3)$$

Equation (3) holds for both individuals, A and B. The first term on the right-hand side is just the marginal rate of substitution (MRS). Remember, at the Pareto optimum, $MRS^A = MRS^B$. Also, what is given to one consumer must be taken away from another, so that $dx^A = -dx^B$.

These relations allow us to develop an alternative expression for dU^B/dU^A. Using expression (3), we can write

$$\frac{dU^B/\left(\partial U^B/\partial x_2\right)}{dU^A/\left(\partial U^A/\partial x_2\right)} = \frac{\dfrac{\partial U^B/\partial x_1}{\partial U^B/\partial x_2}\, dx_1^B + dx_2^B}{\dfrac{\partial U^A/\partial x_1}{\partial U^A/\partial x_2}\, dx_1^A + dx_2^B} \qquad (4)$$

$$= \frac{MRS^B\, dx_1^B + dx_2^B}{MRS^A\, dx_1^A + dx_2^A}$$

Let $Z_1 = MRS^B = MRS^A$; $Z_2 = dx_1^B = -dx_1^A$; $Z_3 = dx_2^B = -dx_2^A$. Also, remember that the first-order conditions for utility maximization gives $\partial U/\partial x_2 = \lambda P_2$, for both individuals, so expression (4) can be rewritten as follows:

$$\frac{dU^B/\lambda^B p_2}{dU^A/\lambda^A p_2} = \frac{Z_1 Z_2 + Z_3}{Z_1(-Z_2) + (-Z_3)}$$

$$\frac{dU^B/\lambda^B}{dU^A/\lambda^A} = -1$$

$$\frac{dU^B}{dU^A} = -\frac{\lambda^B}{\lambda^A}. \qquad (5)$$

Expression (5) shows that the slope of the UPF is equal to the ratio of the marginal utilities of income. Substituting this result in expression (1) shows that the bliss point is realized by equalizing the social marginal utility of income across all individuals:

$$\frac{\partial W/\partial U^A}{\lambda^A} = \frac{\partial W/\partial U^B}{\lambda^B}. \qquad (6)$$

Expression (6) provides a justification for lump-sum income redistribution programs that are analogous to Bentham's arguments. But now, we need to look at the effects of redistribution on social utility of income rather than on individual utility of income. With Bergson-Samuelson, we wish to move income from individuals with low marginal social utility of income to individuals with higher marginal social utility of income, until marginal values are equated. Under the Bentham approach, social welfare was maximized by moving income from individuals with low marginal utility of income to individuals with higher marginal utilities of income.