– Ag Econ/Econ/MAP 514–

PUBLIC GOODS-OPTIMAL OUTPUT

Consider a commodity world containing private goods and one public good. The consumption of the private goods can be represented as

$$\sum_{h=1}^{n} x_i^h = X_i \quad ,$$

where the subscript i denotes the commodity and the superscript h denotes the particular consumer of a population of n. Each consumer consumes a public good G, where $G^h = G$ for all h consumers.

Suppose we wish to maximize a social welfare function of the following form:

$$\max \quad W\left(U^1\left[X^1, G\right], \ldots U^h\left[X^h, G\right], \ldots U^n\left[X^n, G\right]\right)$$

Here the subscripts are suppressed and each X should be interpreted as a vector of private commodities.

Consumption possibilities are constrained by the inputs available for production. Assuming these inputs are also commodities, the production function can be represented as follows:

$$F(X, G) = 0.$$

To see the optimum conditions form the Lagrangean:

$$\mathcal{L} = W + \lambda(0 - F)$$

The first-order conditions are determined as follows:

$$\frac{\partial \mathcal{L}}{\partial X_i^h} = \frac{\partial W}{\partial U^h} \frac{\partial U^h}{\partial X_i^h} - \lambda \frac{\partial F}{\partial X_i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_h \frac{\partial W}{\partial U^h} \frac{\partial U^h}{\partial G} - \lambda \frac{\partial F}{\partial G} = 0$$

These first-order conditions imply the following relations:

$$\frac{\frac{\partial W}{\partial U^h} \frac{\partial U^h}{\partial X_i}}{\sum_h \frac{\partial W}{\partial U^h} \frac{\partial U^h}{\partial G}} = \frac{\frac{\partial F}{\partial X_i}}{\frac{\partial F}{\partial G}}$$

The above condition says that the social marginal rate of substitution between private and public goods is equal to the marginal rate of transformation between private and public goods. Put another way, the ratio of consumer values equals the ratio of marginal costs.

The distinction of the above formulation is related to the consumer valuation of the public good--with a public good, the appropriate "price" is determined by summing marginal utilities across all consumers. This implies a vertical summation of consumer demand curves, rather than the horizontal summation used for private goods.