The Measurement of Producer Surplus

Profit Maximization. Let p = output price, $q(x_i)$ = production function, x_i = input i, and w_i = price of input i.

Optimization Problem

$$\max \quad \pi = p\,q\left(x_i\right) - \sum_i w_i x_i - \text{Fixed Cost} \tag{1}$$

First-Order Conditions

$$\frac{\partial \pi}{\partial x_i} = p \frac{\partial q}{\partial x_i} - w_i = 0 \tag{2}$$

Second-Order Conditions

$$D = \begin{vmatrix} \dfrac{\partial^2 \pi}{\partial x_1^2} & \cdots\cdots & \dfrac{\partial^2 \pi}{\partial x_1\, \partial x_n} \\ \vdots & & \vdots \\ \dfrac{\partial^2 \pi}{\partial x_n\, \partial x_1} & \cdots\cdots & \dfrac{\partial^2 \pi}{\partial x_n^2} \end{vmatrix}$$

Principal minors, D_{ij}, have sign $(-1)^{i+j}$, if solution is a maximum.

Input Demands

By implicit function theorem, first-order conditions (2) can be rearranged so that an input demand function is generated:

$$x_i = x_i^*\left(w_i, p\right) \tag{3}$$

Producer Surplus (PS)

PS = Revenues - Variable Costs

$$PS = pq(x_i[p,w],\ldots,x_n[p,w]) - \sum_i w_i x_i(p,w)$$

$$\frac{\partial PS}{\partial p} = q(x_i[p,w]) + p\sum_i \frac{\partial q}{\partial x_i}\frac{\partial x_i}{\partial p} - \sum_i w_i \frac{\partial x_i}{\partial p}$$

$$= q(\quad) + \left(p\sum_i \left[\frac{\partial q}{\partial x_i} - w_i\right]\right)\frac{\partial x_i}{\partial p}$$

Since $p\dfrac{\partial q}{\partial x_i} = w_i,$ (by first-order conditions)

$$\frac{\partial PS}{\partial P} = q(x_i[p,w],\ldots x_n[p,w])$$

$$\Delta PS = \int_{p^1}^{p^2} q(x_i[p,w])\,dp$$

This measure is the area behind the supply curve, and between the two prices. No path-dependence problems arise in the calculation of PS, because the cross-partials of the supply function, $\partial q(x_i)/\partial p_j$ are equal (by Young's Theorem).