Euler's Theorem and the Equivalence Between Input Costs and Forgone Benefits

Start with a homogeneous production function,

$$(1) \qquad x = f(L_1, L_2, \ldots)$$

where X is the output, and the L_i are the inputs.

Homogeneity means that increasing each input level by a factor of t leads to a t^r increase in output; the exponent r is the degree of homogeneity.

$$f(tL_1, tL_2, \ldots) = t^r f(L_1, L_2, \ldots)$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial tL_1}\frac{\partial tL_1}{\partial t} + \frac{\partial f}{\partial tL_2}\frac{\partial tL_2}{\partial t} + \ldots = rt^{r-1} f(L_1, L_2, \ldots)$$

$$\frac{\partial f}{\partial tL_1} L_1 + \frac{\partial f}{\partial tL_2} L_2 + \ldots = rt^{r-1} f(L_1, L_2, \ldots)$$

The homogeneity condition holds for all values of t. Let t = 1 to get

$$(2) \qquad \frac{\partial f}{\partial L_1} L_1 + \frac{\partial f}{\partial L_2} L_2 + \ldots = rf(\qquad) = rX$$

Under constant returns to scale (linear homogeneity), the value of r is 1. In this case, equation (2) becomes

$$(3) \qquad \frac{\partial f}{\partial L_1} L_1 + \frac{\partial f}{\partial L_2} + \ldots = X$$

Multiply equation (3) by the price of output, P, to generate Euler's Theorem:

$$(4) \qquad \left(P\frac{\partial f}{\partial L_1}\right) L_1 + \left(P\frac{\partial f}{\partial L_2}\right) L_2 + \ldots = PX$$

The expressions in brackets are the marginal value products of the inputs. Under competitive conditions, these values equal the factor prices, w. Substitute to get

$$(5) \quad w_1 L_1 + w_2 L_2 + \ldots = PX$$

Thus, costs of inputs = value of output.

In this case, the value of forgone output is exactly measured by the costs of inputs. So CBA can just consider input costs in the evaluation of a project. The analyst does not need to identify the "forgone project", a difficult (impossible?) task.

The equivalence between input costs and forgone benefits is lost, however, when the forgone project belongs to an industry with economies of scale. With positive economies of scale, $r > 1$, and equation (2) becomes

$$(6) \quad w_1 L_1 + w_2 L_2 + \ldots = r\,PX$$

In this case, the costs of inputs overstate the value of forgone output.