Eric Monke
Ag Econ/Econ/MAP 514

Distributional Weights in the Social Welfare Function

Assume a Bergson-Samuelson social welfare function. Individual utilities are fully measurable and comparable. The SWF is an ordinal function. So welfare change measurement is computed by calculating individual utilities under the alternative states, and then summing these according to the SWF.

To measure these changes, we assume that utility can be written as a function of expenditure

$$W(u_1, u_2, \ldots, u_H) = W(f_1(e_1), f_2(e_2), \ldots f_H(e_H)) \qquad (1)$$

These expenditure functions may need to be standardized to reflect differences among individuals in the ability to convert expenditure into utility (see Boadway and Bruce, pp. 274-275).

Assume that the SWF is strongly separable, so that it can be written as the sum of individual utilities. Further, assume a particular functional form, the isoelastic form (Boadway and Bruce, Chapter 6, pp. 141-142):

$$W = \sum_{i=1}^{H} \frac{(u_i)^{1-\rho}}{1-\rho} \qquad (2)$$

The value of ρ indicates aversion to inequality. For $\rho = 0$, the SWF becomes the Benthamite utilitarian function. As $\rho \rightarrow$ infinity, the SWF becomes the Rawlsian maxi-min SWF.

The marginal social utility of each individual can be written as follows:

$$\frac{\partial W}{\partial u_i} = \frac{1}{(1-\rho)} (1-\rho)\, u_i^{(1-\rho-1)} = u_i^{-\rho} \qquad (3)$$

For $\rho = 0$, marginal social utilities are the same (=1) for all individuals. If $\rho > 0$, then marginal social utility is declining in u_i.

A second assumption is that individual utility functions are identical, and take the following form:

$$u_i = f(e_i) = \frac{e_i^{1-\epsilon}}{1-\epsilon} \,. \qquad (4)$$

The marginal utility of individual income can then be written as follows:

$$\frac{\partial u_i}{\partial e_i} = \frac{\partial f}{\partial e_i} = \frac{1}{(1-\epsilon)} (1-\epsilon)\, e_i^{(1-\epsilon-1)} = e_i^{-\epsilon} \qquad (5)$$

If $\epsilon > 0$, then the marginal utility of income declines as income (e) increases.

A change in social welfare is measured by differentiation of expression (1):

$$dW = \sum_{i=1}^{H} \frac{\partial W}{\partial u_i} \frac{\partial f_i}{\partial e_i}\, de_i \qquad (6)$$

Substitute expressions (3) and (5) into (6) to get an expression:

$$dW = \sum_i u_i^{-\rho} e_i^{-\epsilon}\, de_i$$

$$= \sum_i \left(\frac{e_i^{1-\epsilon}}{1-\epsilon}\right)^{-\rho} e_i^{-\epsilon}\, de_i$$

$$= \sum \left(\frac{1}{1-\epsilon}\right)^{-\rho} e_i^{(1-\epsilon)(-\rho)-(1-\epsilon)+1}\, de_i$$

$$= \sum_i K e_i^{-\sigma}\, de_i \qquad (7)$$

The term $K e_i^{-\sigma}$ represents the marginal social utility of person i's income. It depends on ρ (society's aversion to inequality) and $(1-\epsilon)$ (the individual's elasticity of marginal utility of income).

Now consider the welfare effect of a price change. In this case, de_i is the compensating variation from a price change, or

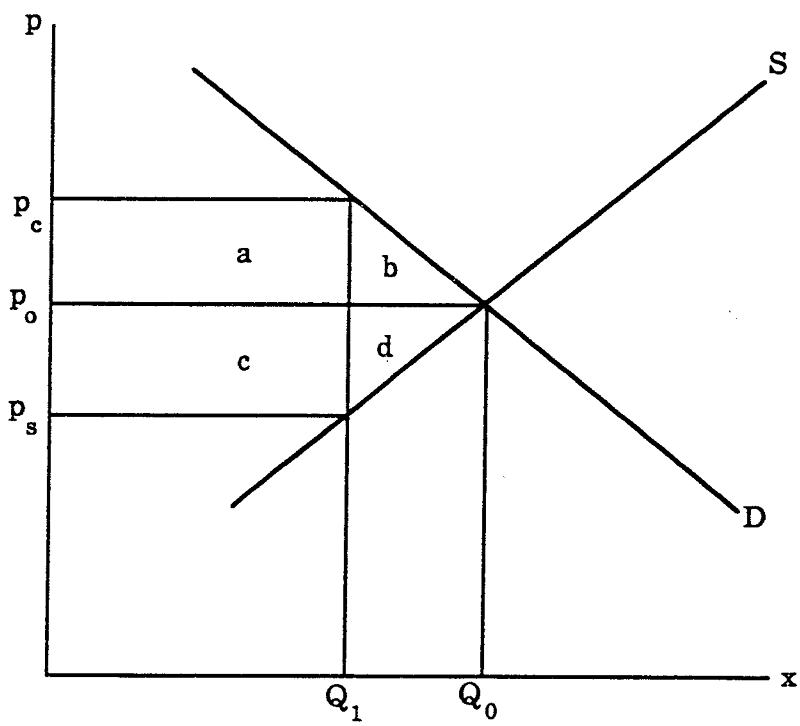
$$d e_i = - x(p, u_i)\, dp$$

The change in social welfare becomes

$$dW = -\sum_i K e_i^{-\sigma} x^i\, dp$$

$$= - R\, x\, dp \qquad (8)$$

where R is the "distribution characteristic" of commodity x. $R = \dfrac{\sum_i K e_i^{-\sigma} x^i}{x}$. The R's represent the distributional weights that are applied to modify the measures of consumer's and producer's surplus. Consider the effect of a sales tax, for example. Initially price is p_o, but the tax causes consumer price to increase to p_c and producer price to decline to p_s.



In the unweighted approach, the welfare measures were as follows:

Consumer effect	= - (a + b)
Producer effect	= - (c + d)
Government revenue effect	= + (a + c)
Net welfare effect	= - (b + d)

With the social welfare function results, the correct welfare measure is altered:

Consumer effect	=	$-R_c\ (a + b)$
Producer effect	=	$-R_s\ (c + d)$
Government revenue effect	=	$+R_t\ (a + c)$
Net welfare effect	=	?

The net effect of the price change on social welfare is now ambiguous. If R_c and R_s are relatively small compared to R_t, the price change could result in a net welfare gain.